Lesson 40

Perimeter and Area When Dimensions are Changed

TAKS Grade 9 Objective 8 (8.10)(A)

A transformation of a figure in which all of the dimensions of the figure are multiplied by the same scale factor is called a **dilation**. If the scale factor is greater than 1, the new figure is larger than the original figure. This type of dilation is called an **enlargement**. If the scale factor is less than 1, the new figure is smaller than the original figure. This type of dilation is called a **reduction**. Any dilation of a figure results in changes to the perimeter and area of the figure.

New Vocabulary

- dilation
- enlargement
- reduction
- scale factor

Effect of Dilation on Perimeter

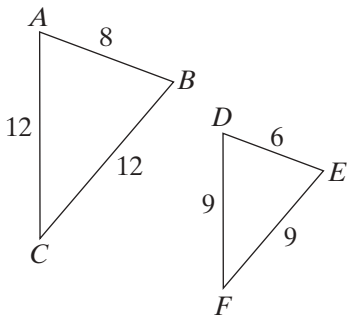
Whenever a figure is dilated by a scale factor, the perimeter of the figure changes according to the same scale factor.

Vocabulary *TIP*

When a figure is dilated, the lengths of the corresponding sides of the old figure and the new figure are in proportion. This proportion is called the **scale factor**.

EXAMPLE 1

$\triangle ABC$ is reduced by a scale factor of $\frac{3}{4}$ to produce $\triangle DEF$. How does this dilation affect the perimeter of the triangle?



Perimeter of $\triangle ABC = 8 + 12 + 12 = 32$

Perimeter of $\triangle DEF = 6 + 9 + 9 = 24$

When $\triangle ABC$ is reduced by a scale factor of $\frac{3}{4}$, the perimeter of the triangle is also reduced by the same scale factor, because: $\frac{24}{32} = \frac{3}{4}$.

Quick Check 1

1a. A hexagon with a perimeter of 24 cm is enlarged by a factor of 1.75. What is the perimeter of the new hexagon?

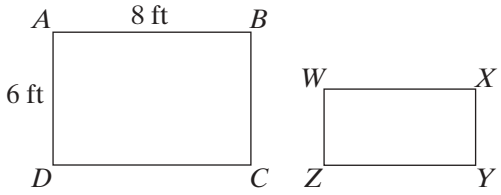
1b. A square with a perimeter of 14 in. is reduced to a square with a perimeter of 7 in. What is the scale factor of this reduction?

Effect of Dilation on Area

When a figure is dilated by a scale factor of $\frac{a}{b}$, the area of the figure is dilated by a scale factor of $\frac{a^2}{b^2}$.

EXAMPLE 2

Rectangle $ABCD$ is reduced by a scale factor of $\frac{1}{2}$ to produce rectangle $WXYZ$. What is the area of rectangle $WXYZ$?



Step 1 Find the area of rectangle $ABCD$.
$6 \text{ ft} \times 8 \text{ ft} = 48 \text{ ft}^2$

Step 2 Find the square of the scale factor.
$(\frac{1}{2})^2 = \frac{1}{4}$

Step 3 Multiply the area of rectangle $ABCD$ by the square of the scale factor to get the area of rectangle $WXYZ$.
$48 \text{ ft}^2 \times \frac{1}{4} = 12 \text{ ft}^2$

The area of rectangle $WXYZ$ is 12 ft^2.

TAKS TIP

When a figure is dilated by a scale factor of $\frac{a}{b}$, it means that the two figures are similar and the ratio of the dimensions of the new figure to the dimensions of the original figure is $\frac{a}{b}$. The ratio of the perimeter of the new figure to the perimeter of the original figure is $\frac{a}{b}$. The ratio of the area of the new figure to the area of the original figure is $\frac{a^2}{b^2}$.

Quick Check 2

2a. A pentagon with an area of 8 cm^2 is enlarged by a scale factor of $\frac{3}{2}$. What is the area of the new pentagon?

2b. A square with an area of 36 square inches is reduced to a square with an area of 9 square inches. What is the scale factor of this reduction?

TAKS Tune-Up Name_____Class_____Date_____

1 A circle has an area of 32 square inches. The circle is dilated by a scale factor of $\frac{1}{2}$ to produce a new circle. What is the area of the new circle?

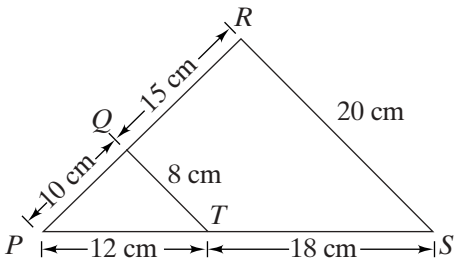
A 4 square inches

B 8 square inches

C 16 square inches

D 64 square inches

2 $\triangle PQT$ is similar to $\triangle PRS$.



Which best represents the ratio of the area of $\triangle PQT$ to the area of $\triangle PRS$?

F $\frac{2}{3}$

G $\frac{2}{5}$

H $\frac{4}{9}$

J $\frac{4}{25}$

3 A company that makes square tiles would like to make a square tile that has 4 times the area of their smaller tiles. If their smaller tiles have a perimeter of 16 inches, what will be the perimeter of their larger tile?

A 8 in.

B 32 in.

C 64 in.

D 256 in.

4 A parallelogram is dilated by a scale factor of $\frac{1}{4}$ to produce a new parallelogram. Which of the following best describes the relationship of the area of the new parallelogram to the area of the original parallelogram?

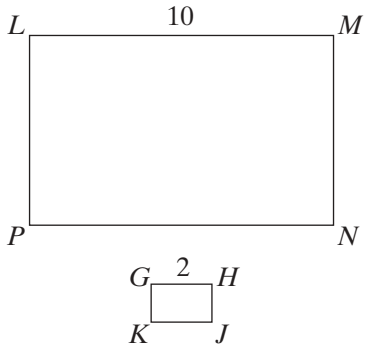
F The area of the new parallelogram is $\frac{1}{16}$ the area of the original parallelogram.

G The area of the new parallelogram is $\frac{1}{8}$ the area of the original parallelogram.

H The area of the new parallelogram is $\frac{1}{4}$ the area of the original parallelogram.

J The area of the new parallelogram is $\frac{1}{2}$ the area of the original parallelogram.

5 Rectangle *GHJK* is a reduction of rectangle *LMNP*. If the area of rectangle *GHJK* is 2 in.^{2}, what is the area of rectangle *LMNP*?



A 5 in.^{2}

B 10 in.^{2}

C 25 in.^{2}

D 50 in.^{2}